

# Thermodynamics for two flavor QCD\*

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We conclude our analysis of the  $N_t = 6$  equation of state (EOS) for two flavor QCD, first described at last year's conference. We have obtained new runs at  $am_q = 0.025$  and improved runs at  $am_q = 0.0125$ . The results are extrapolated to  $m_q = 0$ , and we extract the speed of sound as well. We also present evidence for a restoration of the  $SU(2) \times SU(2)$  chiral symmetry just above the crossover, but not of the axial  $U(1)$  chiral symmetry.

## 1. INTRODUCTION

The equation of state (EOS) is essential for phenomenological models of heavy-ion collision experiments that seek to detect the quark-gluon plasma (QGP). It is also of cosmological interest since the strongly interacting matter present in the early universe might have appeared in such a state.

Following an initial EOS calculation at  $N_t = 4$  by some of us [1], we reported preliminary results for  $N_t = 6$  in Melbourne [2]. Since then, we have completed runs at  $am_q = 0.025$  and extended runs at  $am_q = 0.0125$  to smaller step size. With these additional simulations, we are able to fit our data to empirical expressions, based on standard sigma model assumptions with an  $O(4)$  or mean field critical point at zero quark mass. The fits allow an extrapolation to  $am_q = 0$  and

permit a smooth interpolation of the data, so, for example, the speed of sound can be determined. We have also extended the  $am_q = 0.0125$  data to higher temperature, roughly 250 MeV, where a plateau in the energy density is now evident.

We refer the reader to Refs. [1,2] for details of our method. There we describe how the pressure and energy density are found from derivatives of the partition function with respect to the gauge coupling and bare quark mass and a nonperturbative beta function.

## 2. SIMULATIONS

The analysis requires both hot ( $12^3 \times 6$ ) and cold ( $12^4$ ) lattices. Each hot (cold) simulation is at least 1800 (800) time units long after equilibration. Near the crossover region the simulations were extended to more than 3000 units for the hot lattices. The range of gauge couplings and masses corresponds roughly to physical tem-

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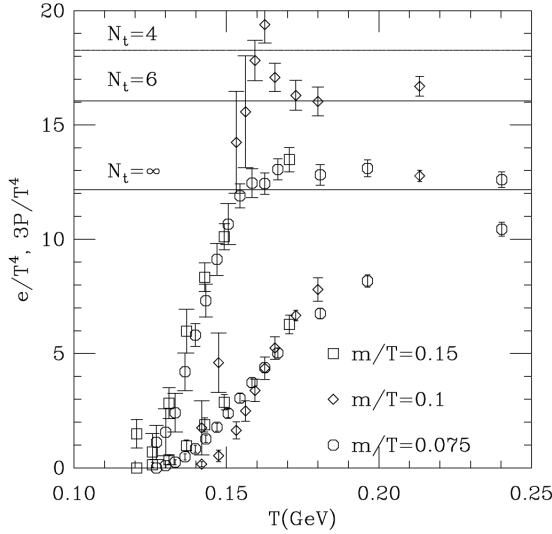


Figure 1. The EOS. The diamonds indicate an earlier result on  $N_t = 4$  lattices.

peratures,  $0 < T < 250$  MeV (based on the  $\rho$  mass [1]), and mass ratios,  $0.3 < m_\pi/m_\rho < 0.7$ . We have also performed simulations at various  $R$  algorithm step sizes  $\Delta t$  to extrapolate results to  $\Delta t = 0$ . For instance, extrapolating the step size from 0.005–0.01 to zero produces a cumulative change in the pressure of 5% at  $6/g^2 = 5.6$ .

### 3. RESULTS

In Fig. 1 we compare the  $N_t = 6$  EOS with the  $N_t = 4$  result and the continuum and lattice Stefan-Boltzmann laws. Here we have converted to a physical temperature scale. There is an apparent large finite size effect which is expected from the free lattice theory. For  $N_t = 6$  we see that the results depend weakly on the quark mass. From the location of the maximum in the slope of  $\langle \square \rangle$  or  $\langle \bar{\psi}\psi \rangle$  with respect to  $6/g^2$ , we conclude that the pseudocritical temperature of the transition is roughly 140 MeV (for both  $am_q = 0.025$  and 0.0125). Figure 1 shows that the energy density is already substantial at this point. At high temperature the energy density has leveled off dramatically and its expected approach to the free lattice result is slow.

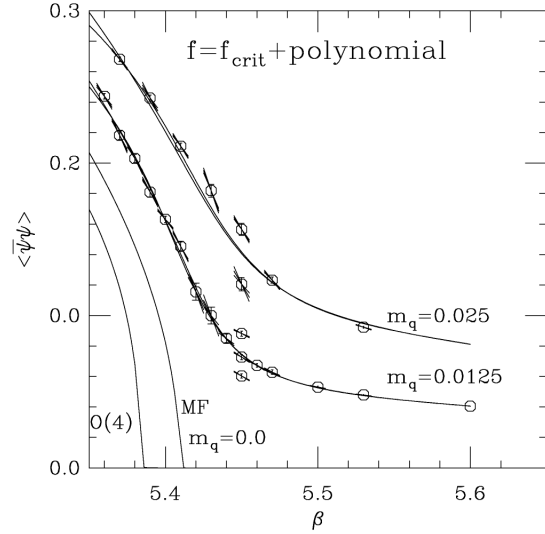


Figure 2. Fit to  $\bar{\psi}\psi$  and its derivative with respect to  $6/g^2$ . The difference between O(4) and mean field is not discernible with the present data. Extrapolation to  $m_q = 0$ , however, gives a different critical coupling.

To obtain a smooth interpolation of the above results and extrapolate to  $m_q \approx 0$  (the physical value), we assume a second order phase transition at  $m_q = 0$  (but see Ukawa's review[3] for cautionary remarks). The nonanalytic part of the free energy obeys the scaling relation

$$f_{\text{crit}}(t, h) = b^{-d} f_{\text{crit}}(b^{y_t} t, b^{y_h} h) \quad (1)$$

where  $t$  and  $h$ , the usual scaling variables, are proportional to  $6/g^2 - (6/g^2)_c$  and  $am_q$ , respectively. Both the critical exponents,  $y_h$  and  $y_t$ , and the scaling function  $f_{\text{crit}}(t, h)$  are universal. Two flavor QCD has long been expected to be in the same universality class as the 3d O(4) Heisenberg magnet [4]. While the O(4) critical exponents are known [5], the scaling function is not. Therefore we have performed new simulations of the O(4) magnet to determine the scaling function. Recently it has been argued that the transition may be mean field [6], for which the entire form of the scaling part of the free energy is known [7].

In Fig. 2 we show preliminary fits of  $\bar{\psi}\psi$  and its derivative to the mean field and O(4) scal-

ing functions plus a polynomial in  $am_q$  and  $6/g^2$ . For the quark masses used in our simulations, the fits are indistinguishable. However, the respective extrapolations to  $m_q = 0$  are quite different. In Fig. 2 the mean field and O(4) critical couplings correspond to *zero mass* temperatures  $T_c \approx 150$  and 160 MeV, respectively. When  $\langle \square \rangle$  and its derivative are included in the fits, these values are shifted down by about 10 MeV. The above indicates that present lattice simulations may still be too far from the scaling region and smaller quark masses are required to see the true scaling behavior.

An extrapolation of the EOS to  $m_q = 0$  is shown in Fig. 3. It is compared with the  $am_q = 0.0125$  result, which reproduces the data reasonably well. Note that here we fit  $\langle \square \rangle$ ,  $\langle \bar{\psi}\psi \rangle$  and their derivatives with respect to  $6/g^2$  simultaneously. The appearance of the bump in the energy density just after the transition is probably an artifact of the extrapolation (at  $m_q = 0$ , the corresponding region of  $6/g^2$  lies below the values of the coupling where we have done simulations). From Fig. 3 we again see a weak dependence on the quark mass. In Fig. 4 we show the speed of sound calculated from this fit. The low temperature part of the curve is not trustworthy since the derivatives of the energy density and pressure are poorly known in this region. In fact, we expect the hadron gas below the transition to have a nonzero speed of sound, which then dips down at the transition.

#### 4. CHIRAL SYMMETRY RESTORATION

Sigma model analogies suggest that in the chiral limit of zero quark mass in the quark-gluon plasma phase, the  $SU(2) \times SU(2)$  chiral symmetry is exact, but the anomalous  $U(1)_A$  axial symmetry may remain broken[4]. Early efforts to assess the status of these symmetries looked for the expected degenerate multiplets in screening masses. However, because these studies did not include difficult-to-measure quark-line disconnected graphs, they were inconclusive[8]. The large data sample generated in the EOS study makes it feasible to reexamine this ques-

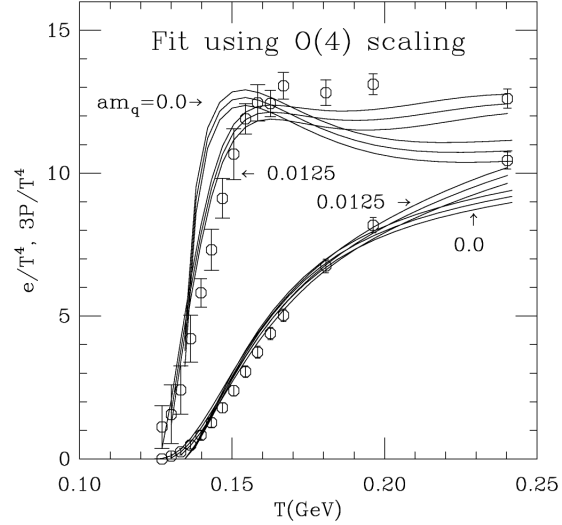


Figure 3. The EOS extrapolated to  $am_q = 0.0$ . The fit and data at  $am_q = 0.0125$  are also shown. The triplet of fit curves at both masses, as labeled, indicates the central value and one standard deviation spread resulting from the statistical uncertainty in the fit.

tion. Knowing the answer gives important insight into the mechanics of the phase transition. The Columbia group has also pursued this question [9], the Bielefeld group has reported preliminary results [10], as has Lagae [11]

To address this question we focused on susceptibilities for the following hadron channels:  $f_0$  (also known as  $\sigma$  or  $\bar{\psi}\psi$ ),  $a_0$  ( $\delta$ ), and  $\pi$ . For example the  $f_0$  susceptibility is constructed from the point-source correlator as follows:

$$\chi_{f_0} = \int d^4x \langle f_0(x) f_0(0) \rangle - \langle f_0(0) \rangle^2 \quad (2)$$

From these susceptibilities, we define the order parameters

$$\chi_{U(1)} = \chi_\pi - \chi_{a_0}; \quad \chi_{SU(2)} = \chi_{f_0} - \chi_\pi. \quad (3)$$

If the order parameter is nonzero in the chiral limit, the corresponding symmetry is broken.

The  $f_0$  susceptibility can be decomposed into quark-line-connected and disconnected parts as  $\chi_{f_0} = \chi_{\text{conn}} + \chi_{\text{disc}}$ . The connected part is identified with  $\chi_{a_0}$ , so is obtained from the point hadron correlator using traditional methods. The

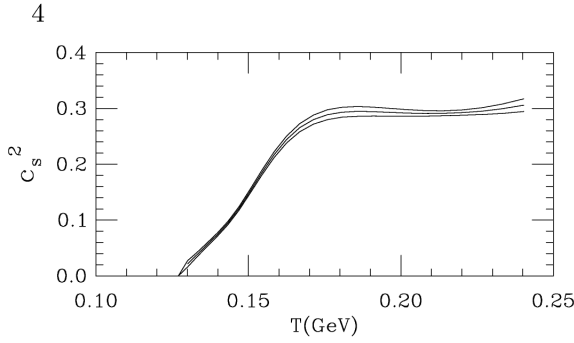


Figure 4. The speed of sound for  $am_q = 0.0125$ . The three curves indicate the central value and the one standard deviation spread resulting from the statistical uncertainty in the fit.

disconnected part is the variance from configuration to configuration of the space-time volume average of  $f_0 = \bar{\psi}\psi$ . The latter is conventionally measured from  $\text{Tr}(\not{D} + m_q)^{-1}$ , which we estimated using multiple random sources on each configuration. Finally, the pion susceptibility  $\chi_\pi$  can also be obtained from  $\langle f_0 \rangle / m$ .

For this study we fixed  $6/g^2 = 5.45$  and varied the quark mass from  $am_q = 0.0075$  to  $0.025$ , corresponding to the high temperature phase. The crossover to the low temperature phase occurs at a slightly higher mass, and the chiral limit corresponds to a temperature of approximately  $1.2T_c(ma = 0)$ .

Preliminary results are shown in Fig. 5. In the two-flavor staggered fermion scheme the fermion determinant is rigorously even in the quark mass. In consequence the order parameters are also even in  $am_q$ . For  $T > T_c$  there are no known infrared singularities, even in the chiral limit. Thus we have indicated a linear extrapolation in  $(am_q)^2$ , leading to a result consistent with the sigma model scenario: a restoration of  $SU(2) \times SU(2)$  but not of  $U(1)_A$  (approximately  $2\sigma$ ).

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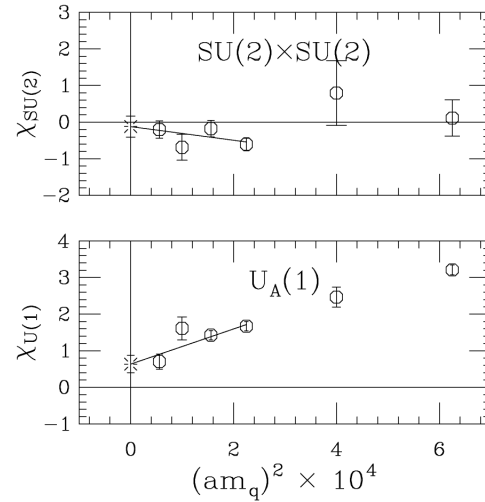


Figure 5. Chiral symmetry order parameters for  $T > T_c$ , extrapolated to zero quark mass (burst symbol). The two high mass points closest to the crossover were omitted from the fit.

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